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Microwave Sound Waves in Solids

MARDEN H. SEAVEY, JR., Electromagnetic Radiation Laboratory, AFCRL

If a ferromagnetic solid is exposed to the magnetic field from another such solid, as for example a large laboratory electromagnet, the lattice structure of the solid usually becomes distorted. If the magnetic field is then rapidly varied, for example by adding a microwave magnetic field to it, mechanical or sound waves are set up in the solid. Quantized units of such sound waves are known as phonons, just as photons are quantized units of electomagnetic energy.

AFCRL has been engaged in fundamental studies of microwave phonons in ferromagnetics for the past two years. Important accomplishments, both of a theoretical and experimental nature, have been achieved. These contributions place AFCRL at the forefront of the new research field of microwave phonon generation, a field of widespread and growing interest.

A central reason for the interest is that high frequency phonons provide a distinctive new tool for the exploration of many phenomena in the solid state. Hypersonic waves can penetrate far more deeply into conducting or semiconducting media than EM microwave energy of comparable frequency. Thus important properties of many solids, which until now have remained unmeasurable, can be determined by the use of hypersonic waves.

But apart from the use of phonons as tools for the analysis of conductors and semiconductors they offer several device applications. Since the velocity of the phonon is much slower (by 10^5) than the photon, appreciable times delays occur in relatively short crystal

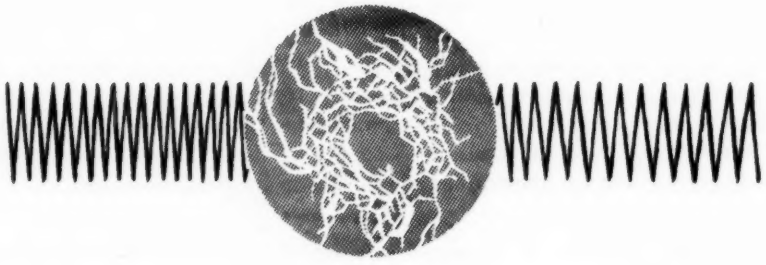
lengths. Thus, the possibility of a solid state microwave delay line arises. Phonons may also provide the key to microwave modulation of laser light beams.

Much of the research in microwave phonon generation over the past several years has concentrated on a quartz as a phonon source. Although AFCRL explorations include the study of quartz and other piezoelectrics, the basic AFCRL contributions in the field center about two classes of materials: thin magnetic films and yttrium iron garnet.

A recent paper prepared by Marden H. Seavey, Jr., of AFCRL, reports on his work on the generation of phonons by thin magnetic films. The mechanism responsible for phonon generation in the case of thin films is the same as it is in all ferromagnetics, namely magnetostriction. This phenomenon relates magnetization to mechanical stress. When a magnetization vector processes in a ferromagnetic medium, as it does during ferromagnetic resonance, a mechanical stress follows the precession. Discontinuities in the precession create discontinuities in mechanical stress. In regions where the precession is discontinuous, or changing rapidly, sound waves at a microwave frequency, namely microwave phonons, originate. Such regions occur at the surfaces of a thin film since the uniform magnetization precession is discontinuous there.

Thin films have a number of advantages over quartz as a transducing media. One of these is the low losses that occur in propagating phonons from one material to another--losses as low as 3db in thin films as compared with losses of about 20db in quartz, at X-band frequencies. Other advantages are potentially higher transducing efficiency and greater flexibility in generating transverse and longitudinal phonons.

The generation mechanisms of yttrium iron garnet, although they also involve magnetostriction, are somewhat more subtle than in the thin films case. The inherently low loss properties of certain yttrium iron garnet materials, however, offer great promise for controlled phonon generation. AFCRL is currently exploring an exciting new technique for the amplification of phonons in such materials.



the Cullowhee Conference

OLIVER A. SHAW, Major, USAF
Mathematics Division, AFOSR

For generations many applications of mathematics have been found in Physics. From being spectacular and shocking to the point of threatening the annihilation of all established religion, the scientific advances brought about by these tools have become expected and routine, to the point of threatening nothing worse than the destruction of the human race.

Specialized branches of mathematics such as Probability Theory and Applied Statistics have found most important applications in Agricultural Science, Government and Industry; chief among these being perhaps, the Design and Analysis of Experiments concerned with efficient choices, the area of Quality Control and Reliability, and Actuarial Theory.

In recent times the use of mathematics has begun to appear in biological fields. Although these appearances are more sporadic, they remain diverse and disconnected due to (let us hypothesize) the fact that few biologists know an appreciable amount of mathematics and few mathematricians are at all concerned with Biology. The importance of biological phenomena, encompassing life itself, cannot be over emphasized. It appears obvious that a greater effort should be made to use mathematics in the study of biology. To this aim, several research centers have been devoting considerable time and study for several years. The need for training creative people in a combination of the two disciplines has become apparent.

H. L. Lucas, and his staff at North Carolina State College, decided in 1960 that the time was now ripe to collect the ideas of those who had been studying this problem and to make them available to all. Accordingly, a conference was organized which took place in mid-August of 1961 at Western Carolina College, Cullowhee, N. C. This conference was jointly sponsored by the Institute of Statistics of N. C. State College, the Division of General Medical

Sciences of the NIH and the Mathematics Division of AFOSR. It would be inappropriate in this brief account to repeat the list of a hundred or more participants -- such a list is published at the end of the single volume "Proceedings" entitled "The Cullowhee Conference on Training in Biomathematics" just published. It might be appropriate to abstract a small, almost representative, sample -- to wit -- A. F. Bartholomay, Harvard Medical School; W. J. Dixon, UCLA; T. L. Lincoln, Yale Medical School; M. Lloyd, Oxford U; W. H. Hatheway, The Rockefeller Foundation, Bogota, Columbia; G. W. Rasch, The Danish Institute for Educational Research; N. Rashevsky, U of Chicago; H. R. Van derVaart, Institute for Theoretical Biology, Leiden, Netherlands; and S. Yamamoto, Nara Medical College, Japan.

The Proceedings, in addition to a brief history and state of the art summary contains discussions of applications of the following topics as related to biology: (1) Principles of Applying Mathematics. (2) Differential Equations and Classical Analysis. (3) Linear and Relational Methods. (4) Probability, Information Theory and Statistics. (5) Computers, Simulators, and Cybernetics. and, (6) Training Requirements.

We feel that this is a most timely volume.

MEASUREMENT OF **STARFISH** *RADIATION*

LUDWIG KATZ, Ionospheric Physics Laboratory, AFCRL

Since 27 October 1962, there has been little decrease in the intensity of trapped radiation injected into the Van Allen Belt by the July 9 nuclear detonation in the Pacific.

This conclusion, based on data from satellites passing through the radiation belt, was reported by AFCRL scientist, Ludwig Katz, at a recent press conference held at AFCRL.

Data was taken from a specially instrumented Air Force satellite designed to measure the enhanced radiation. Katz reported that an apparent state of equilibrium has been reached in which there is either no decay in radiation, or so little change as to be within the narrow margin of experimental errors of observation.

The 9 July detonation at an altitude of about 250 miles above the

equator placed a large number of high energy particles into one of the lower shells of the earth's magnetic field. These particles are produced by radioactive decay of nuclear debris. Following an initial fairly rapid decrease, there has been an apparent leveling off in the decay rate. The present level of radiation within the particular magnetic shell in which the detonation took place is considerably higher than the radiation level prior to the shot, it was reported.

The AFCRL scientist also reported that the satellite acquired data on Russian nuclear detonations. These Russian shots were detonated at a higher geomagnetic latitude than the U. S. shot. The bulk of the injected particles from the Russian tests lasted only a few days.

It was explained that the rapid decay of the trapped particles from the Russian detonations may have resulted from the fact that these particles in their motion in the magnetic shell reached closer to the earth. The probability that particles are removed increases with their closeness to the earth's surface where the atmosphere causes particle loss.

The measurement and analysis of the particles artifically injected by the U. S. and Russian nuclear detonations, may shed considerable light on the mechanism of particle loss and replacement which maintains equilibrium in the natural Van Allen Belts.

The AFCRL satellite was instrumented and launched in a record time of two and one-half months following the 9 July shot. Responsibility for the design and construction of the scientific instrumentation for the satellite was assigned to the Air Force Camgridge Research Laboratories (AFCRL), in Bedford, Mass. The satellite achieved an apogee of 3000 nautical miles and a perigee of 115 nautical miles in a 71 degree orbital inclination. The apogee chosen reached through the normal proton belt into the lower L shells of particular interest at equatorial levels and continued on to higher L shells of the magnetic field as the satellite reached polar latitudes.

The selection of a low perigee, reaching down into the ionosphere, was planned in order to delineate the lower edge of the particle trapping region, in the hope of being able to determine the conditions responsible for eventual radiation decay.

The instrumentation was designed to measure the nature of trapped particles, their intensity, their energy spectrum, and their angular distribution. While the responsibility for the overall design of of the payload rested with AFCRL, other scientific groups in the country contributed instruments to the payload.

the history of a *Phase Diagram*

THOMAS J. ROCKETT, Metallurgy and Ceramics Research Laboratory, ARL

The second most abundant material in the earth's crust is silica. It not only occurs as the major constituent of vast sand deposits but it is a major component of many igneous, sedimentary and metamorphic rocks. Technologically it is one of our most important materials. It is the most important material used in the glass industry; it has wide use for such ceramic materials as refractory bricks and porcelains and as sandstone it is an important building material. Also, many of the colored varities serve as semi-precious gems.

Silica occurs in so many different forms that a cursory glance would lead one to conclude that there is little systematic relationship between all these materials which are chemically SiO_2. It is the purpose of phase equilibria studies to determine the pyrochemical reactions between all of the phases in a chemical system and present phase diagrams which graphically represent the conditions for the stability of a given phase. Since silica is a common material in extremely wide use and since it has been used by man since earliest times, one might well suspect that it is a material upon whose reactions there would be reasonably good agreement. Such is not the case. There exists today, among chemists, ceramists, and mineralogists a lively controversy regarding the stability of the phases of silica. This laboratory has been able to throw considerable light upon this problem. It can best be understood if one traces, historically, the development of our knowledge about silica.

One of the forms occurs as striking crystals which have a trigonal trapezohedral symmetry, and these have often been used as ornamental stones. In the Book of Exodus there is a reference to a material used to decorate breastplates. The mineral referred to as "adamus" was probably a form of silica. The next recorded reference to this crystalline form of silica is in Theophrastus' famous treatise on stones which was written in 314 B. C. He described the crystalline form of silica and called it, "cristallos" which was the

Greek word meaning ice. The Greeks believed that all matter could be classified as either fire, earth, ether, or water and observing these transparent crystals concluded that they were, indeed, water which had been frozen at an extremely low temperature and this "deep-freeze" resulted in their durability and peculiar form.

No new ideas were contributed by Pliny or other Romans and we find no written record of silica again until the Middle Ages. The books, known as "lapidaries", such as Albertus Magnus's "De Mineralibus" listed the occurrance of some of the silica minerals but told us little regarding their nature. In 1546, Agricola in his treatise "De Natura Fossilium", first classified minerals on the basis of physical properties. Among the minerals listed was the crystalline form of silica to which he gave the name quartz.

During the next three hundred years as science grew to one of the major areas of human knowledge, the understanding of the nature of quartz advanced with it. Nicolaus Steno, the father of crystallography, used quartz in his experiments which lead to the first observation of the principle of constancy of interfacial angles in 1669. Lavoisier, in 1787 ventured the opinion that quartz was an oxide of an unknown metal, and thirty-six years later, Berzelius isolated and described that metal which became known as silicon. With improved chemical analysis, it was recognized in the nineteenth century, that the glass sands, as well as, the ubiquitous, anhedral, glassy rock-forming mineral, and the well defined crystals known as quartz were chemically identical. When the microscope became an important tool for mineralogists, it was further discovered that the crystal structures of all of the above forms of SiO_2 were identical. They are all, therefore, the one phase of silica known as quartz.

In 1875, J. Willard Gibbs published his famous treatise in which he defined the phase rule. Using his principles, it is possible to draw a phase diagram for a one component system which represents the stability relations between the various phases. In such a diagram, the theoretical vapor pressure of a given phase is plotted against temperature. While the vapor pressure is not actually known, a starting vapor pressure is assumed and the diagram is plotted in accordance with the two following principles: that the vapor pressure of a phase increases with temperature and that for any given temperature, the most stable phase has the lowest vapor pressure. If, then, we were to draw a phase diagram in 1875 which represented the system SiO_2 it would look like figure 1.

In figure 1 the following thermochemical reactions are represented: (1) Quartz $\rightleftharpoons$ liquid silica (2) Liquid silica $\longrightarrow$ glass (Le Chatelierite) (3) Glass $\longrightarrow$ quartz.

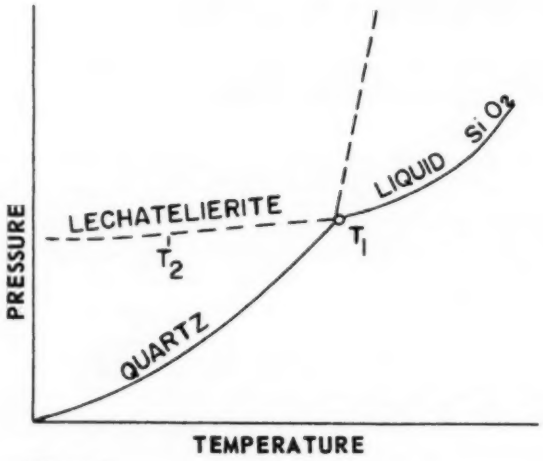


Figure 1

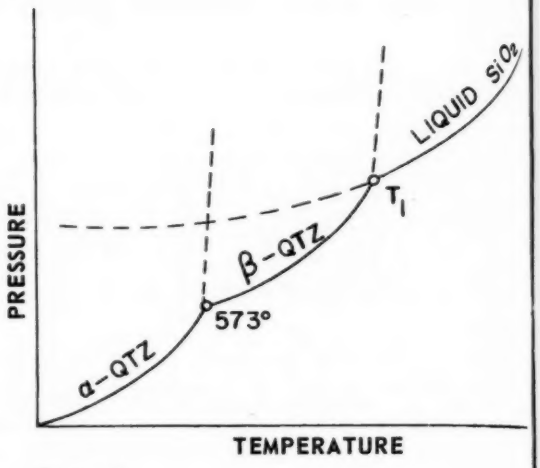


Figure 2

Reaction (2) is a metastable reaction and its vapor pressure, therefore, is represented by a dashed line over the temperature range of its metastable existence. At the melting temperature of quartz, fused silica should crystallize upon cooling. If, however, the liquid is cooled past the crystallization temperature the glass is formed and stays indefinitely at room temperature. If the glass is reheated to T_2 then reaction (3) takes place. Such a reaction from the metastable state to the stable state is known as a monotropic inversion because at the given temperature the reaction is not reversible. Since this glass has been observed in nature, it has the mineral name lechatelierite. One of its occurrences is as long, irregular, tubular masses formed when lightning strikes sand.

Toward the end of the nineteenth century Le Chatelier discovered that quartz exists in two polymorphic forms. He observed a large break in the thermal expansion curve at 573°C. The incorporation of this reaction, α-quartz $\rightleftharpoons$ β-quartz, into the phase diagram would lead to figure 2.

The next phase of silica was discovered by Rose in the lava beds of Cerro San Cristobal near Pachuca, Mexico. He found some octahedral crystals which analysed as SiO_2. Since quartz has a trigonal structure, it could never form octahedra and therefore he named this new phase cristobalite after its locality. Figure 3 shows a phase diagram which

incorporates this new phase. Geologically, lavas are associated with high temperature and, hence, cristobalite is a given a higher temperature stability. Also, since it was discovered at room temperature, it can be retained in a metastable state. Using these facts, one might draw figure 3.

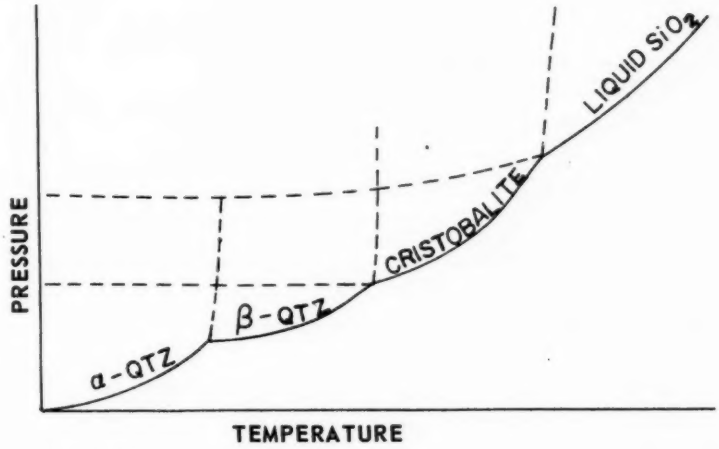


Figure 3

Shortly after the discovery of cristobalite yet another phase was discovered from the same locality. This mineral, because of its occurrence in crystals of three twins was called, tridymite. Just where this new phase would fit on the diagram could not be determined without experimental data.

The work was performed in 1913 by Clarence N. Fenner at the Carnegie Institute's Geophysical Laboratory in Washington, D. C. In his truly classic work he published a diagram for the system which is shown, with minor revisions, in figure 4. This work stood, unchallenged, as the basic work in silica chemistry for forty years. From this diagram, we can list the following reactions: Stable phases; enantiotropic (i. e., reversible) inversions.

(4) α-quartz $\overset{573^{\circ}}{\rightleftharpoons}$ β- quartz.

(5) β-quartz $\overset{870^{\circ}}{\rightleftharpoons}$ β-tridymite.

(6) β-tridymite $\overset{1470^{\circ}}{\rightleftharpoons}$ β-cristobalite.

(7) β-cristobalite $\overset{1723^{\circ}}{\rightleftharpoons}$ liquid silica.

Metastable phases: enantiotrophic (i.e. reversible) inversions.

(8) β-quartz $\overset{1100^{\circ}}{\rightleftharpoons}$ β-cristobalite.

(9) α-cristobalite $\overset{260^{\circ}}{\rightleftharpoons}$ β-cristobalite.

(10) β′-tridymite $\overset{163^{\circ}}{\rightleftharpoons}$ β-tridymite.

(11) α-tridymite $\overset{117^{\circ}}{\rightleftharpoons}$ β′-tridymite.

Metastable, Monotropic, (i.e. irreversible) inversions.

(12) silica glass $\xrightarrow{900 - 1700^{\circ}}$ β-cristobalite.

(13) β-cristobalite $\xrightarrow{870 - 1470^{\circ}}$ β-tridymite.

(14) β-tridymite $\xrightarrow{1680}$ liquid silica.

(15) β-quartz $\xrightarrow{1400}$ liquid silica.

Since World War II, improved instrumentation has allowed investigators to examine the behavior of silica under high pressures. This work has led to the discovery of three new phases of silica: keatite, coesite, and stishovite. While the first phase seems to be a metastable transition phase, the others have stability fields at high pressures. A diagram encorporating these phases is given in figure 5.

From Figure 5, we can now add to our list of silica reactions the following stable pressure sensitive inversions:

(16) quartz $\overset{P,T}{\rightleftharpoons}$ liquid (17) tridymite $\overset{P,T}{\rightleftharpoons}$ quartz

(18) cristobalite $\overset{P,T}{\rightleftharpoons}$ quartz. (19) quartz $\overset{P,T}{\rightleftharpoons}$ coesite

(20) coesite $\overset{P,T}{\rightleftharpoons}$ liquid. (21) coesite $\overset{P,T}{\rightleftharpoons}$ stishovite

(22) stishovite $\overset{P,T}{\rightleftharpoons}$ liquid.

Figure 5

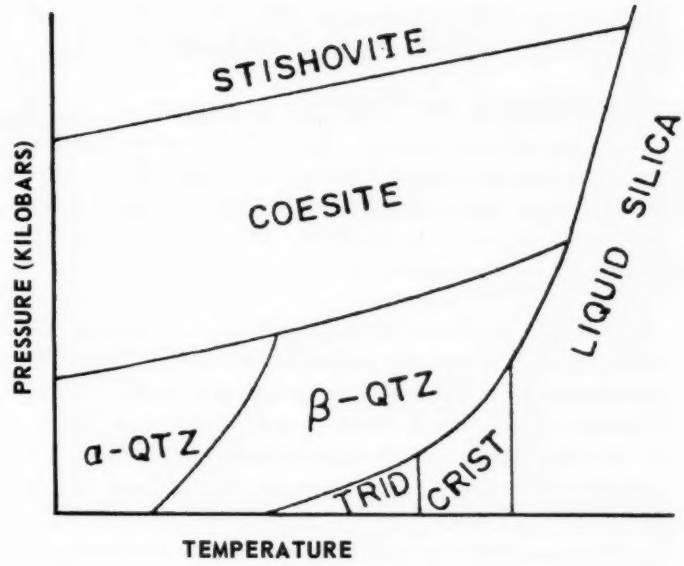
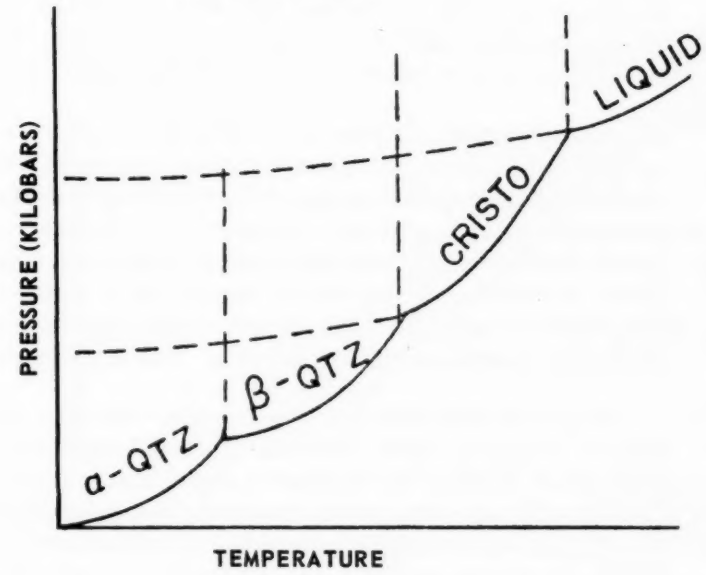


Figure 6



There are two more phases of pure SiO_2 which have been reported in recent years. The first has been called silica "W" and is an extremely unstable phase whose crystal structure collapses just by contact with water vapor. This phase then, can not be represented on a diagram. The other phase, first reported in 1876, but only substantiated within the last year is the mineral, melanophlogite.

This cubic form seems to be a low temperature modification but no thermal studies have been reported as yet.

In 1956, O. W. Florke, a scientist from the Max Planck Institute for Silicate Research, published some findings from which he concluded that Fenner's phase diagram does not adequately describe the behavior of silica. He introduced a new diagram, the essence of which is shown in figure 6, in which he eliminated the phase tridymite from the system.

All of the reactions listed in this paper for silica, with the exception of the α β reactions, are extremely sluggish, some requiring many days to go to completion. To speed these reactions, Fenner used a flux which he claimed served only as a catalyst and in no way entered into the reacting phases. Florke tried to prepare tridymite from pure SiO_2 and found that it apparently could not be done without a flux. He concluded, therefore, that the flux was not just a catalyst but reacted with the pure silica to form a binary phase, tridymite. The amount of foreign ion necessary to form this phase has been placed by various American, French, and German scientists in the range 0.1 and 1 weight percent of the foreign oxide.

At this point, in the development of silica studies, the Metallurgy and Ceramics Research Laboratory at the Aeronautical Research Laboratories began investigating the reactions, at elevated temperatures, of silica with boron, with boron oxide, and with other oxides. In the presentation of the work we had three choices; avoid the tridymite problem completely, give an either/or presentation of the work, or carry out experimental work in an attempt to settle the problem conclusively. The third course was chosen.

The task involved synthesizing tridymites by the use of various fluxes, studying their thermal stabilities and purifying them by an extraction method or by electrodialysis. If tridymite did not exist as pure silica, then upon removal of the foreign ions from its structure, the material should break down into the stable cristobalite or quartz polymorph. We not only produced tridymites having less than 0.01 weight percent of the foreign ion allegedly necessary for stabilization, but we further showed that this low impurity tridymite is more stable than the high impurity material. The results of this work will be published shortly and while we do not claim to have said the last work on tridymite, we have shown that Figure 6 is an erroneous diagram and that Fenner's diagram does, indeed, represent the reactions of silica at ambient pressures.

PANG-BANG control systems

BERNARD S. MORGAN, JR., Captain, USAF
Applied Mathematics Division, AFOSR

A pang - bang control system results from the specification of time optimal control of a physical system. Such a control is an on - off control - one which requires full power on or off or full thrust right or left. Thus a pang - bang (also referred to as bang - bang) situation occurs.

Optimal control theory refers to a method of designing control systems wherein the traditional preliminary stability investigation is completely bypassed. Here one specifies the nature and states of the physical system; the nature of the possible control forces and torques which could be used; the criterion of performance to be maximized or minimized. One then proceeds via sophisticated mathematical techniques to find the control system design which optimizes the specified performance criterion.

Optimal control theory when applied to a physical system requires the knowledge of the instantaneous physical state of the system. The physical state can be specified by the measurement of a fixed number of quantities which characterize the system for the purposes considered. As an example, the state of a rigid body in space can, for certain purposes be defined by a set of 14 numbers: namely, the position and velocity vectors (each having 3 components) of the center of mass, the angular orientation (3 components), angular velocity vector (3 components), and finally the body 's mass and temperature.

Let us now return to how the traditional stability investigation is bypassed in optimal control theory. This is done, initially, by prescribing that there exsists a class of control laws which are stable in the sense that they bring the actual physical state indistinguishably close to the desired state after some period of time. Fortunately, certain control laws that are stable and also physically realistic do exist. A few of these are those that: (a) take the least amount of time (the pang - bang control); (b) use the least amount of fuel (or the least possible expenditure of energy); (c) cause the least possible temperature use. Unfortunately, classical stability and control theory, whose techniques and procedures are based on the theory of linear differential equations, cannot be used to design systems which

are optimum in the sense required by these practical control considerations. The reason is that actual and optimal control systems are inherently nonlinear. In fact, only through recent advances in such fields as the calculus of variations has progress been made in the solution of the mathematical representations of optimal control theory.

The Air Force is especially interested in optimal control theory because of its possible application to the high performance requirements of space vehicle attitude control systems as well as to the more conventional autopilots of improved winged aircraft. This interest is typified at the basic research level by AFOSR support of the control theory research of Professor Chang, Dr. LaSalle of RIAS, and Professor Flugge-Lotz of Stanford. Here the intent is total generality, mathematical thoroughness and rigor, and a drive for deeper understanding of the basic abstract theory. However, the theory should be applicable to real-life physical systems. Recent research by Professor Chang (1, 2, 3,) has been so directed. One paper (1) removed one essential but impractical condition in the present optimal control theory. In pang - bang control the rudder is assumed to be inertialess. Its position can be changed instantly from -a to a. Yet this is never true in actual ships and planes.

This problem however can be considered as a special case of a more general problem, that of optimal control in bounded phase space. For instance, in controlling an airplane, the elevator and ailerons are limited in both speed and displacement. One way to remove the multiple limits on the movements of the controls is to consider the velocities of the elevator and ailerons as controls only, and to regard the displacements as states together with the other dynamical variables (states) of the airplane. Then the states representing the displacements are bounded. For such a problem a necessary condition for optimal control has been obtained that has been shown to be sufficient under certain conditions. In practical terms this condition means that the minimum time control for a system with multiple saturations limits is a pang-bang system.

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